Q.48. What is the net flux through a closed surface due to a charge located outside it ?

Ans. Zero.

Q.49. An arbitrary surface encloses an electric dipole of dipole moment 20×10^{-6} C-m. What is the electric flux through this surface ? **[A.I. 2012, NCERT Exemplar]**

Ans. Zero, because net charge within the closed surface is zero.

Q.50. If the radius of the Gaussian surface enclosing a surface is halved, how does the electric flux through the Gaussian surface change ? **[A.I. 2008]**

Ans. As the charge enclosed by the Gaussian surface remains unchanged, hence the electric flux also remains unchanged.

Q.51. If Coulomb's law had $\frac{1}{r^3}$ dependence instead of $\frac{1}{r^2}$, would Gauss's theorem had been still valid ?

Ans. No, because Gauss's theorem is essentially based on inverse square law dependence of Coulomb's law.

Q.52. What is the electric flux through a cube of side 1 cm which encloses an electric dipole ? **[Delhi 2015]**

Ans. Total flux is zero because net charge enclosed within the cube is zero.

Q.53. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (*i*) the inner surface, and (*ii*) the outer surface ? **[NCERT Exemplar]**

Ans. Due to presence of a charge Q at the centre of spherical cavity, a charge $-Q$ is induced on the inner surface of shell and a charge $+Q$ is induced on the outer surface as shown in Fig. 1.27.

(*i*) ∴ Surface charge density on inner surface $\sigma_1 = \frac{-Q}{A_1} = \frac{-Q}{4\pi R_1^2}$

(*ii*) ∴ Surface charge density on the outer surface $\sigma_2 = \frac{+Q}{A_2} = +\frac{Q}{4\pi R_2^2}$.

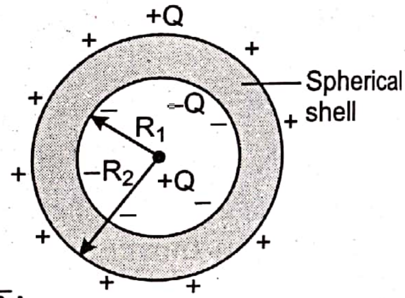

Fig. 1.27

Q.54. A sphere S_1 of radius r_1 encloses a total charge Q. If there is another concentric sphere S_2 of radius r_2 (> r_1) and there be no additional charge between S_1 and S_2, find the ratio of electric flux through S_1 and S_2.

Ans. $\frac{\phi_2}{\phi_1} = \frac{q_1/\epsilon_0}{q_2/\epsilon_0} = \frac{q_1}{q_2} = \frac{Q}{Q} = 1.$ [Since charge within both spheres is same = Q]

(B – I) Short Answer Type Questions (2 marks each)

Q.1. Two charged spherical conductors, each of radius R, are distant d ($d > 2R$). They carry charges $+q$ and $-q$. Will the force of attraction between them be exactly $\frac{q^2}{4\pi\epsilon_0 d^2}$? **[A.I. 2000]**

Ans. No, the force of attraction between charged spherical conductors will be more than $\frac{q^2}{4\pi\epsilon_0 d^2}$. On account of mutual attractive force there will be a redistribution of charges on the spheres as shown in Fig. 1.28. As a result, effective distance between them is reduced and the force increases.

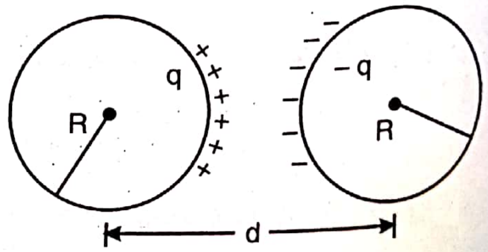

Fig. 1.28

Q.2. Two identical metallic spheres, having unequal opposite charges are placed at a distance 0.9 m apart in air. After bringing them in contact with each other they are again placed at the same distance apart. Now the force of repulsion between them is 0.025 N. Calculate the final charge on each of them. **[Delhi 2002]**

Ans. As two metallic spheres are identical, after bringing them in contact charges on both will be same (say q). As $r = 0.9$ m and $F = 0.025$ N.

Hence from the relation $F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{r^2}$, we have

$$q = \sqrt{4\pi\epsilon_0 \cdot F \cdot r^2} = \sqrt{\frac{0.025 \times (0.9)^2}{9 \times 10^9}} = 1.5\ \mu C.$$

Q.3. Two similarly and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Calculate the net electric force on C. **[A.I. 2003]**

Ans. Let charges on A and B be q each and r be the distance between them. Then as shown in Fig. 1.29(a), we have

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{r^2} = 2 \times 10^{-5}\ \text{N}. \quad ...(i)$$

On touching C with A, charge on A as well as C will be $\frac{q}{2}$. Now as shown in Fig. 1.29(b) net force on C is

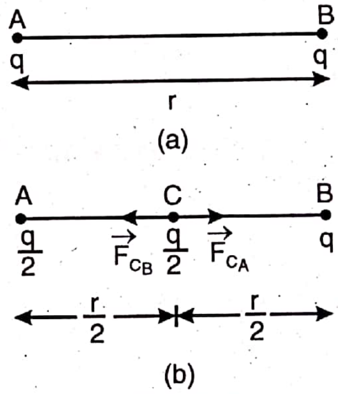

Fig. 1.29

$$F_C = |\vec{F}_{CA} + \vec{F}_{CB}| = F_{CB} - F_{CA} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\left(\frac{q}{2}\right)(q)}{\left(\frac{r}{2}\right)^2} - \frac{1}{4\pi \epsilon_0} \cdot \frac{\left(\frac{q}{2}\right)\left(\frac{q}{2}\right)}{\left(\frac{r}{2}\right)^2}$$

$$= \frac{1}{4\pi \epsilon_0}\left[\frac{2q^2}{r^2} - \frac{q^2}{r^2}\right] = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{r^2} = 2 \times 10^{-5}\ \text{N along } CA.$$

Q.4. The electric field E due to a point charge at any point near it is defined as

$$E = \lim_{q_0 \to 0} \frac{F}{q_0}$$

where q_0 is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q_0 \to 0}$ in this expression ?

Draw the electric field lines of a point charge Q when (i) $Q > 0$, and (ii) $Q < 0$. **[Delhi 2007]**

Ans. In the definition of electric field E we take test charge q_0 as small as possible so that presence of test charge does not affect the electric field at all. That's why we consider $\lim_{q \to 0}$.

Electric field lines of a point charge (i) $Q > 0$, and (ii) $Q < 0$ have been plotted in Figs. 1.18 and 1.19 respectively.

Q.5. Fig. 1.30 shows the electric field lines around three point charges A, B and C.

(a) Which charges are positive ?

[NCERT Exemplar]

(b) **Which charge has the largest magnitude ? Why ?**

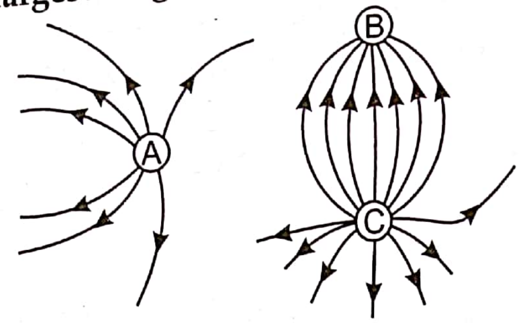

Fig. 1.30

Ans. (a) Charges A and C are positive but charge B is negative.

(b) The charge C has largest magnitude because maximum number of electric field lines are originating from it.

Q.6. Derive an expression for the electric field at a distance r from a point charge q.

Ans. Consider a test charge q_0 placed at a point P situated at a distance r from a point charge q.

$\therefore$ Force experienced by test charge $$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q \cdot q_0}{r^2}$$

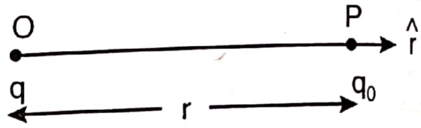

Fig. 1.31

$\therefore$ Electric field at given point $$E = \frac{F}{q_0} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{r^2}$$

Vectorially $$\vec{E} = \frac{q}{4\pi \epsilon_0 r^2} \cdot \hat{r} = \frac{q}{4\pi \epsilon_0 r^3} \vec{r} .$$

Q.7. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$ where r is the distance between the two charges of each pair of charges : (1 μC, 2 μC) and (2 μC, – 3 μC). Interpret the graphs obtained. **[A.I. 2011]**

Ans. Force between two charges $q_1 = 1\ \mu C = 1 \times 10^{-6} C$ and $q_2 = 2\ \mu C = 2 \times 10^{-6}$ C separated by a distance r,

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{r^2} = \frac{18 \times 10^{-3}}{r^2} \text{N}$$

and force between two charges $q_1' = 2\ \mu C = 2 \times 10^{-6}$ C and $q_2' = -3\ \mu C = -3 \times 10^{-6}$ C separated by a distance r, the force

$$F' = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1' q_2'}{r^2} = \frac{9 \times 10^9 \times (2 \times 10^{-6}) \times (-3 \times 10^{-6})}{r^2}$$

$$= -\frac{54 \times 10^{-3}}{r^2} \text{N}$$

Graph showing variation of force F and F' versus $\frac{1}{r^2}$ is shown in adjoining Fig. 1.32. From the graph we note following points :

(i) Both the graphs are straight line graphs. As force F is repulsive, its graph is in 1st quadrant. But force F' is attractive and the graph is in 4th quadrant.

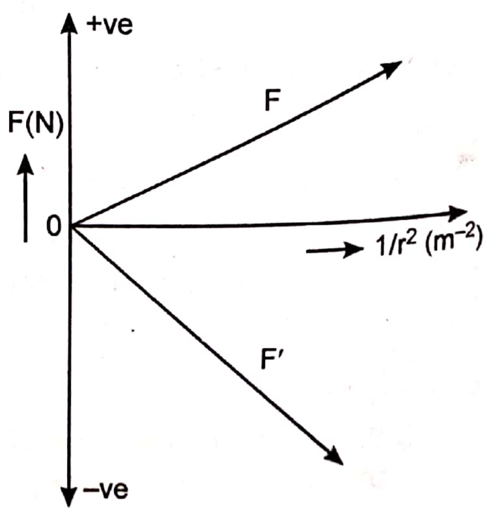

Fig. 1.32

(*ii*) As $|\vec{F'}| = 3|\vec{F}|$, hence slope of $F - \frac{1}{r^2}$ graph for second pair of charges is 3 times as compared to that for first pair of charges.

Q.8. Two charges $+q$ and $-q$ are kept at $(-x_1, 0)$ and $(x_1, 0)$ respectively in the x-y plane. Find the magnitude and direction of the net electric field at the origin (0, 0). **[A.I. 2009]**

Ans. Let two charges of $+q$ and $-q$ be kept at points A and B having coordinates $(-x_1, 0)$ and $(x_1, 0)$ as shown in Fig. 1.33 in the x-y plane.

Net electric field at origin point $O = \vec{E} = \vec{E_A} + \vec{E_B}$

But $\vec{E_A} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{x^2}$ along AOB and $\vec{E_B} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{x^2}$ along OB. Hence net electric field at point O

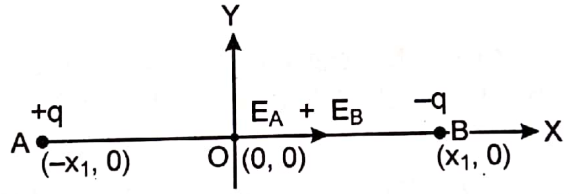

Fig. 1.33

$$\vec{E} = 2\times\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{x^2} = \frac{q}{2\pi\epsilon_0 x^2} \text{ along } AOB.$$

Q.9. Two charges Q and $-3Q$ are placed fixed on x-axis separated by a distance 'd'. Where should a third charge $2Q$ be placed such that it will not experience any force ? **[NCERT Exemplar]**

Ans. Let charges Q and $-3Q$ be placed at points A and B separated by a distance d. The third charge $2Q$ should be placed at point P on extension of line BA nearer the smaller charge of Q at a distance x from it. As net force on $2Q$ charge is zero.

$$\therefore \quad \vec{F_p} = \vec{F_A} + \vec{F_B} = 0 \quad \text{or} \quad \vec{F_A} = -\vec{F_B}$$

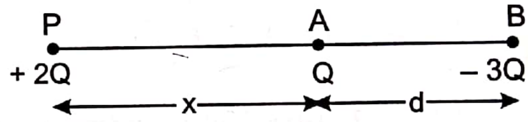

Fig. 1.34

$$\Rightarrow \quad \frac{1}{4\pi\epsilon_0}\cdot\frac{Q(2Q)}{x^2} = -\frac{1}{4\pi\epsilon_0}\cdot\frac{(-3Q)(2Q)}{(x+d)^2}$$

$$\Rightarrow \quad \frac{1}{x^2} = \frac{3}{(x+d)^2} \quad \Rightarrow \quad x = \frac{d}{(\sqrt{3}-1)}$$

Q.10. Two equal charges of $+2\times10^{-16}$ C are kept 20 cm apart in air. At a point midway between them find (*i*) electric field, and (*ii*) force acting on a charge of $+2\times10^{-16}$ C. **[A.I. 2002]**

Ans. (*i*) Electric field at O, $\vec{E} = \vec{E_A} + \vec{E_B}$

Now, $$|\vec{E_A}| = |\vec{E_B}| = \frac{1}{4\pi\epsilon_0}\cdot\frac{2\times10^{-16}}{(0.1)^2}$$

but their directions are mutually opposite.

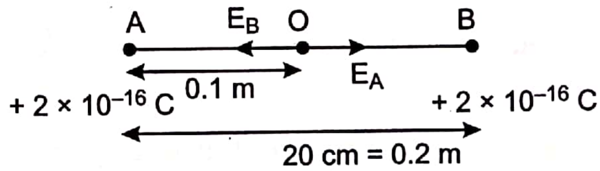

Fig. 1.35

Hence, their vector sum is zero *i.e.*, $\vec{E} = \vec{0}$

(*ii*) Force acting on a charge of $q_0 = 2\times10^{-16}$ C

$$\vec{F} = q_0\vec{E} = 2\times10^{-16}\times0 = 0.$$

Q.11. A particle of mass 10^{-3} kg and charge 5 μC enters into a uniform electric field of 2×10^5 N C^{-1}, moving with a velocity of 20 m s^{-1} in a direction opposite to that of the field. Calculate the distance it would travel before coming to rest. **[Delhi 2012 C]**

Ans. Here mass of particle $m = 10^{-3}$ kg, charge $q = 5\ \mu C = 5 \times 10^{-6}$ C, initial velocity of particle $u = 20$ m s^{-1}, final velocity $v = 0$ and electric field $E = -2 \times 10^5$ N C^{-1}. (The electric field has been taken – ve because its direction is opposite to that of direction of motion of charged particle).

$$\therefore \text{Acceleration of particle } a = \frac{F}{m} = \frac{qE}{m} = \frac{(5\times10^{-6})\times(-2\times10^{5})}{10^{-3}} = -10^{3}\text{ m s}^{-2}$$

From equation $v^2 - u^2 = 2as$, the distance travelled by charged particle

$$s = \frac{v^2 - u^2}{2a} = \frac{(0)^2 - (20)^2}{2(-10^3)} = 0.2\text{ m}$$

Q.12. Define electric field. Write its SI unit. Write the magnitude and direction of electric field due to an electric dipole of length '2a' at the mid-point of the line joining the two charges. **[A.I. 2005]**

Ans. Electric field at a point is defined as the ratio of force experienced by a small positive test charge placed at that point to the magnitude of test charge. Its SI unit is N C^{-1}.

We know that electric field at a point on the equatorial line of a dipole is given by

$$\vec{E} = -\frac{1}{4\pi\epsilon_0}\cdot\frac{\vec{p}}{(a^2 + r^2)^{3/2}}$$

At the mid-point of dipole $r = 0$ and hence

$$\vec{E} = -\frac{1}{4\pi\epsilon_0}\cdot\frac{\vec{p}}{a^3}$$

Its direction is opposite to that of dipole moment $\vec{p}$.

Q.13. Deduce the expression for the electric field $\vec{E}$ due to a system of two charges q_1 and q_2 with position vectors $\vec{r_1}$ and $\vec{r_2}$ at a point P having position vector $\vec{r}$. **[Delhi 2010 C]**

Ans. Let as shown in Fig. 1.36 two point charges q_1 and q_2 be situated at points A and B respectively with position vectors $\vec{r_1}$ and $\vec{r_2}$. Let P be a point having position vector $\vec{r}$. Electric field at P due to charge q_1

$$\vec{E_1} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1}{|\vec{r} - \vec{r_1}|^3}\cdot(\vec{r} - \vec{r_1})$$

and field at P due to charge q_2, $\vec{E_2} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_2}{|\vec{r} - \vec{r_2}|^3}\cdot(\vec{r} - \vec{r_1})$

$\therefore$ Total electric field at point P is

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r} - \vec{r_1}|^3}\cdot(\vec{r} - \vec{r_1}) + \frac{q_2}{|\vec{r} - \vec{r_2}|^3}\cdot(\vec{r} - \vec{r_2})\right]$$

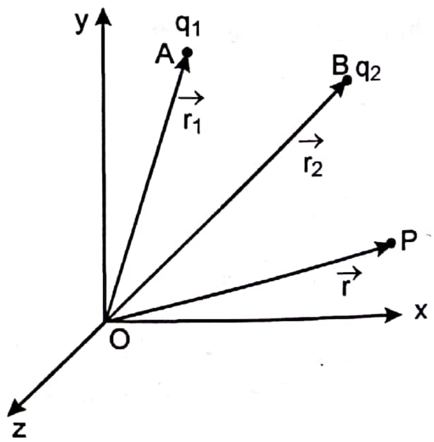

Fig. 1.36

Q.14. A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if $q = -\frac{Q}{4}$. **[A.I. 2005]**

Ans. For equilibrium of system the net force on any charge must be zero. Considering net force on A, we have $\vec{F}_{AC} + \vec{F}_{AB} = 0$

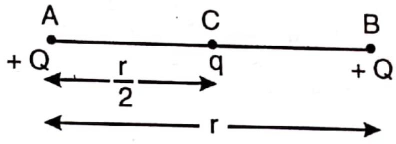

Fig. 1.37

$$\therefore \quad \frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{\left(\frac{r}{2}\right)^2} + \frac{1}{4\pi\epsilon_0}\cdot\frac{Q^2}{r^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}[4q+Q] = 0$$

or $$4q + Q = 0$$

or $$q = -\frac{Q}{4}.$$

Q.15. Derive an expression for the electric field at any point along the axial line of an electric dipole. **[Delhi 2001, 2003]**

Ans. The electric field at a point P on axial line of dipole due to $+q$ charge at B

$$E_B = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(r-a)^2} \text{ along } BP$$

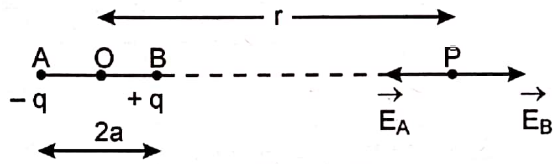

Fig. 1.38

and electric field due to $-q$ charge at A is

$$E_A = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(r+a)^2} \text{ along } PB$$

$\therefore$ Net electric field at point P is given by

$$E = E_B - E_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q\,.\,4ar}{4\pi\epsilon_0(r^2-a^2)^2}$$

$$\Rightarrow \quad E = \frac{2pr}{4\pi\epsilon_0(r^2-a^2)^2},$$ where $p = q.2a$ = dipole moment of given dipole.

For a short dipole or if the point P is situated far away, then $a << r$ and hence

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{2p}{r^3} \text{ along } ABP$$

Vectorially $$\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{2\vec{p}}{r^3}$$

Q.16. An electric dipole is free to move in a uniform electric field. Explain its motion when it is placed **[Delhi 2002]**

(*i*) parallel to the field, and (*ii*) perpendicular to the field.

Ans. (*i*) When an electric dipole is placed parallel to a uniform electric field, net force as well as net torque acting on the dipole is zero and, thus, the dipole remains in equilibrium.

(*ii*) When dipole is placed perpendicular to the field, a torque $\vec{\tau} = \vec{p} \times \vec{E}$ acts on it. Under its influence the dipole executes oscillatory motion about the direction of electric field and finally aligns itself along the direction of electric field.

Q.17. An electric dipole, when held at 30° with respect to a uniform electric field of 10^4 N/C, experiences a torque of 9×10^{-26} N m. Calculate the dipole moment of the dipole.

Ans. Here $E = 10^4$ N/C, $\tau = 9 \times 10^{-26}$ N m and $\theta = 30°$

From the relation $\tau = p\,E \sin\theta$, we have

$$p = \frac{\tau}{E \sin\theta} = \frac{9 \times 10^{-26}}{10^4 \times \sin 30°} = \frac{9 \times 10^{-26}}{10^4 \times 0.5} = 1.8 \times 10^{-29} \text{ C m}$$

Q.18. Three point charges of + 2 μC, – 3 μC and – 3 μC are kept at the vertices *A*, *B* and *C*, respectively of an equilateral triangle of side 20 cm as shown in the Fig. 1.39. What should be the sign and magnitude of the charge to be placed at the mid-point (*M*) of side *BC* so that the charge at *A* remains in equilibrium ? [Delhi 2005]

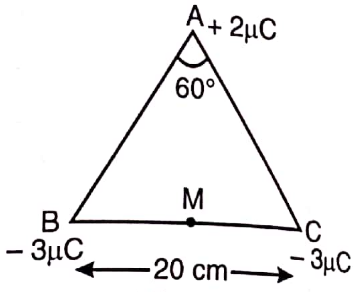

Fig. 1.39

Ans. Charge q on point M should be such that force on A due to it should just nullify the resultant of forces $\vec{F}_{AB}$ and $\vec{F}_{AC}$.

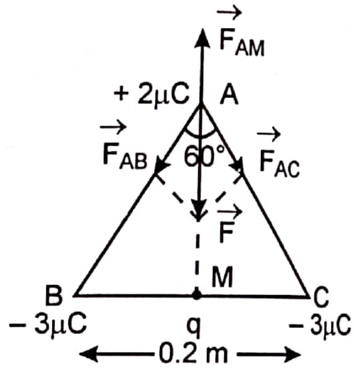

Fig. 1.40

Now
$$F_{AB} = F_{AC} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{(0.2)^2} = 1.35 \text{ N}$$

Resultant of $\vec{F}_{AB}$ and $\vec{F}_{AC}$ inclined at an angle of 60° is

$$\vec{F} = 2\,\vec{F}_{AB} \cdot \cos\frac{\theta}{2} = 2 \times 1.35 \times \cos 30°$$

$$= 2 \times 1.35 \times \frac{\sqrt{3}}{2} = 1.35 \times \sqrt{3} \text{ N along } AM$$

For equilibrium, force $\vec{F}_{AM}$ should be equal and opposite to $\vec{F}$

$$\therefore \quad F = |\vec{F}_{AM}| = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times q}{(AM)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times q}{\left[(0.2)^2 - (0.1)^2\right]}$$

or
$$1.35 \times \sqrt{3} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times q}{0.03}$$

$$\Rightarrow \quad q = \frac{1.35 \times \sqrt{3} \times 0.03}{9 \times 10^9 \times 2 \times 10^{-6}} = 3.93 \times 10^{-6} \text{ C or } 3.93\,\mu\text{C}.$$

Q.19. Three small charges of equal magnitude and same sign lie on the circumference of a circle forming an equilateral triangle. What is the value of electric field at the centre of circle ?

Ans. As shown in Fig. 1.41, electrical fields at the centre of circle due to three charges are E_A, E_B and E_C, whose magnitudes are equal. As these intensities are being represented by three vectors of same magnitude and making an angle of 120° from one another, the resultant field

$$\vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C = \vec{0}$$

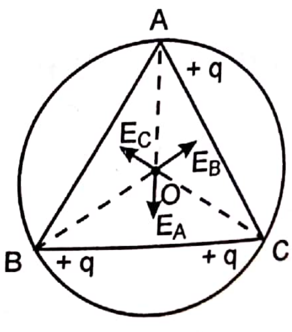

Fig. 1.41

Q.20. Define electric flux. Write its SI units. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases its size, how does the total electric flux coming out of the surface change ? Give reason. [Delhi 2007]

Ans. For electric flux and its SI unit, see Point Numbers 45, 46 and 47 under the heading "Chapter At A Glance".

Even on blowing up of the balloon total electric flux coming out of its surface remains unchanged.

Total flux $\phi_E = \frac{1}{\epsilon_0}(Q)$, where Q = charge enclosed within the closed surface. As charge enclosed remains unchanged, hence electric flux remains unchanged.

Q.21. Define electric flux. Write its SI unit.

A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change. **[A.I. 2009]**

Ans. For electric flux and its SI unit, see Point Number 45 and 46 under the heading "Chapter At A Glance".

The electric flux through the surface remains unchanged at $\phi_E = \frac{q}{\epsilon_0}$ in accordance with Gauss' law, because the charge enclosed within the surface remains unchanged.

Q.22. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **[A.I. 2011]**

Ans. As shown in Fig. 1.42, a thin straight infinitely long conducting wire having linear charge density λ is enclosed by a cylindrical surface of radius r and length l with its axis coinciding with the length of the wire.

As per Gauss's law total electric flux through the entire surface of the cylinder

$$\phi_E = \frac{1}{\epsilon_0} \text{ (charge enclosed within the cylinder)}$$

$$= \frac{1}{\epsilon_0}(\lambda l) = \frac{\lambda l}{\epsilon_0}$$

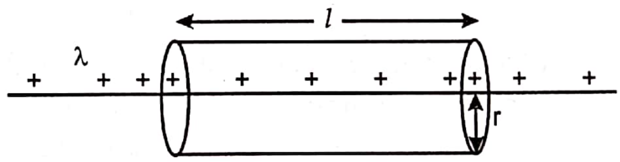

Fig. 1.42

Q.23. State Gauss's theorem in electrostatics.

A charge of 17.7×10^{-4} C is distributed over a large sheet of area 200 m^2. Calculate the electric field intensity at a distance 20 cm from it in air. **[A.I. 2003]**

Ans. For statement of Gauss's theorem, see Point Number 50 under the heading "Chapter At A Glance".

Here $q = 17.7 \times 10^{-4}$ C and surface area $A = 200$ m^2

$\therefore$ Surface density of charge $\sigma = \frac{q}{A} = \frac{17.7 \times 10^{-4}}{200}$

$\therefore$ Electric field at a point near the sheet

$$E = \frac{\sigma}{2\epsilon_0} = \frac{17.7 \times 10^{-4}}{200 \times 2 \times 8.85 \times 10^{-12}} = 5 \times 10^5 \text{ N C}^{-1}$$

The electric field is independent of the distance from the sheet.

Q.24. Two plane sheets of charge densities $+\sigma$ and $-\sigma$ are kept in air as shown in the Fig. 1.43. What are the electric fields at points A and B? **[Delhi 2003]**

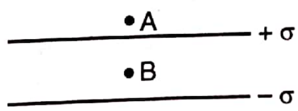

Fig. 1.43

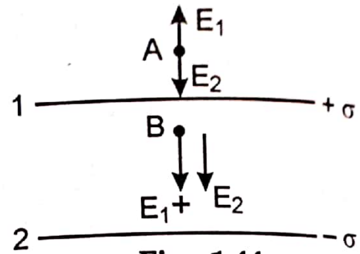

Fig. 1.44

Ans. (*i*) At point A, the electric fields E_1 and E_2 due to sheets 1 and 2 have magnitude $\frac{\sigma}{2\epsilon_0}$ but their directions are mutually opposite as shown in Fig. 1.44. Hence, net electric field at A, $E_A = 0$.

(*ii*) At point B electric fields due to both sheets are in same direction as shown in Fig. 1.44. Moreover

$$|E_1| = |E_2| = \frac{\sigma}{2\epsilon_0}$$

∴ Net electric field at point B

$$E_B = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \text{ normally towards 2nd plate.}$$

Q.25. Given a uniform electric field $\vec{E} = 5 \times 10^3\ \hat{i}$ N C^{-1}. Find the flux of this field through a square of 10 cm on a side whose plane is parallel to the Y-Z plane.

What would be the flux through the same square if the plane makes a 30° angle with the x-axis ? **[Delhi 2014]**

Ans. Here $\vec{E} = 5 \times 10^3\ \hat{i}$ N C^{-1} and area of square $S = 10 \times 10\ cm^2 = 100\ cm^2 = 100 \times 10^{-4}\ m^2 =$ $0.01\ m^2$. Since surface is in Y-Z plane, hence $\vec{S} = 0.01\hat{i}\ m^2$.

$$\therefore \quad \text{Flux } \phi_E = \vec{E}\cdot\vec{S} = ES = (5\times10^3)\times0.01 = 50\ \text{N m}^2\ \text{C}^{-1}$$

When plane of square makes a 30° angle with the x-axis, then area vector $\vec{S}$ subtends an angle $\theta = 60°$ from x-axis and hence new flux will be

$$\phi'_E = ES\cos\theta = (5\times10^3)\times0.01\times\cos 60° = 25\ \text{N m}^2\ \text{C}^{-1}$$

Q.26. An electric flux of – 6 × 10^3 N m^2 C^{-1} passes normally through a spherical Gaussian surface of radius 10 cm, due to a point charge placed at the centre.

(*i*) What is the charge enclosed by the Gaussian surface ?

(*ii*) If the radius of Gaussian surface is doubled, how much flux would pass through the surface ?

Ans. Here electric flux $\phi_E = -6\times10^3\ \text{N m}^2\ \text{C}^{-1}$

(*i*) As $\phi_E = \frac{q}{\epsilon_0}$, hence charge enclosed by the Gaussian surface $q = \epsilon_0 . \phi_E$

$$\therefore \quad q = 8.85 \times 10^{-12} \times (-6 \times 10^3) = -5.31 \times 10^{-8}\ \text{C}.$$

(*ii*) Even on doubling the radius of Gaussian surface the flux passing through the surface will remain unchanged because charge within the surface is even now same as before.

Q.27. S_1 and S_2 are two hollow concentric thin spherical shells enclosing charges Q and $2Q$ respectively as shown in Fig. 1.45.

(*i*) What is the ratio of the electric flux through S_1 and S_2 ?

(*ii*) How will the electric flux through the shell S_1 change, if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air ? **[A.I. 2002, 2014 C]**

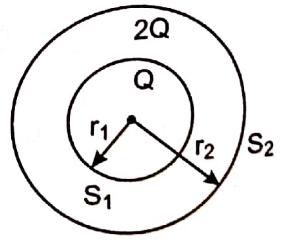

Fig. 1.45

Ans. (*i*) According to Gauss's law electric flux through spherical shell S_1

$$\phi_1 = \frac{1}{\epsilon_0}\cdot Q$$

and flux through outer shell S_2 $\quad \phi_2 = \frac{1}{\epsilon_0}(Q+2Q) = \frac{1}{\epsilon_0}\cdot 3Q$

$$\Rightarrow \qquad \frac{\phi_1}{\phi_2} = \frac{1}{3}$$

(*ii*) When a medium of dielectric constant $K = 5$ is introduced in the space inside the shell S_1 in place of air, flux through S_1 will be modified to

$$\phi_1' = \frac{1}{\epsilon}\cdot Q = \frac{1}{K\epsilon_0}Q = \frac{\phi_1}{K} = \frac{\phi_1}{5}$$ *i.e.*, flux will be reduced to $\frac{1}{5}$th of its previous value.

Q.28. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and another charge $+2Q$ is placed outside the shell at point A at a distance x from the centre $(x > R)$.

Find (*i*) the force on the charge at the centre of the shell and at the point A (*ii*) the electric flux through the shell. **[Delhi 2015]**

Ans. (*i*) The force on the charge $\frac{Q}{2}$ placed at the centre of shell is zero because electric field at point C, inside the shell is zero.

For the purpose of electric force on charge 2Q placed at point A, the charge Q present on spherical shell may be considered to be present at the centre point C. Thus, net charge at C is $\frac{Q}{2}+Q=\frac{3Q}{2}$.

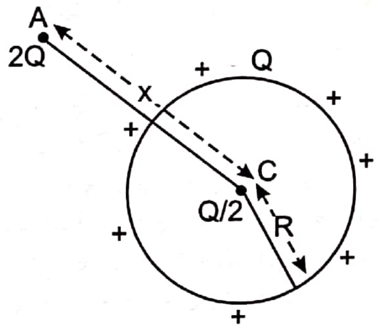

Fig. 1.46

$\therefore$ Force at point A, $\quad F_A = \frac{1}{4\pi\epsilon_0}\cdot\frac{(3Q/2)(2Q)}{x^2} = \frac{1}{4\pi\epsilon_0}\cdot\frac{3Q^2}{x^2}$

(*ii*) Electric flux through the shell $\phi_E = \frac{1}{\epsilon_0}$(charge enclosed) $= \frac{1}{\epsilon_0}\cdot\frac{3Q}{2} = \frac{3Q}{2\epsilon_0}$

Q.29. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge 'Q'. A charge 'q' is placed at the centre of the shell.

(*a*) What is the surface charge density on the (*i*) inner surface, (*ii*) outer surface of the shell ?

(*b*) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell. **[A.I. 2010]**

Ans. (*a*) Let us have a conducting shell of inner radius r_1 and outer radius r_2. When it is given a charge Q, the charge spreads on its outer surface and there is no charge on inner surface of shell.

When a charge q is placed at the centre of the shell, it induces a charge $-q$ on inner surface 1 of the shell and a charge $+q$ on outer surface 2 of the shell.

(*i*) The surface charge density on the inner surface

$$\sigma_1 = \frac{-q}{4\pi r_1^2}$$

(*ii*) The surface charge density on the outer surface

$$\sigma_2 = +\frac{Q+q}{4\pi r_2^2}$$

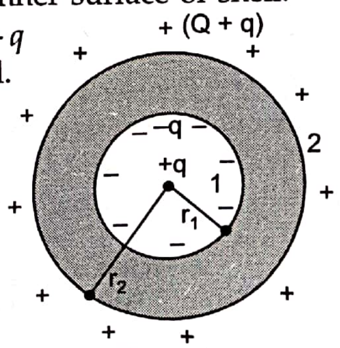

Fig. 1.47

(*b*) For a point lying outside the spherical shell $(x > r_2)$, electric field E is given by

$$E = \frac{\text{total charge on spherical shell}}{4\pi\epsilon_0 x^2} = \frac{Q+q}{4\pi\epsilon_0 x^2}$$

Q.30. A small metallic sphere carrying charge +Q is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_1. **[Delhi 2014 C]**

Ans. As shown in the adjoining figure a charge $-Q$ is induced on the inner surface of the metallic shell and, in turn, a charge $+Q$ is induced on the outer surface of the shell.

The electric field at a point P_1 situated in the cavity at a distance r from the centre of charged small metallic sphere is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} \hat{r}$$

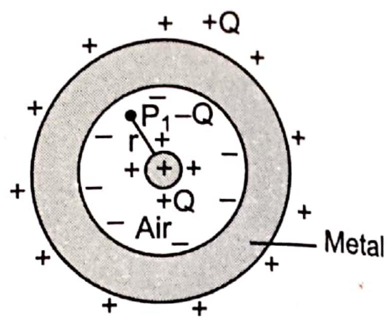

Fig. 1.48

The field is solely due to charge $+Q$ of small metallic sphere and there is no field due to charges induced on the hollow metallic shell.

Q.31. Show that net force acting on an electric dipole held in a uniform electric field is zero. **[A.I. 2012]**

Ans. Consider an electric dipole AB held in a uniform electric field held at an angle θ with the direction of electric field $\vec{E}$. Now force acting on $+q$ charge, $F_1 = qE$ along direction of $\vec{E}$ and force acting on $-q$ charge, $F_2 = qE$ in a direction opposite to that of $\vec{E}$.

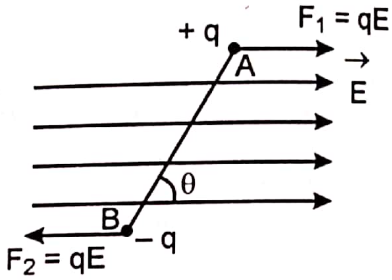

Fig. 1.49

Since F_1 and F_2 are equal but in mutually opposite directions, hence the net translational force acting on the electric dipole will be zero.

Q.32. A certain charge Q is divided into two parts q and $(Q - q)$, which are then separated by a certain distance r. What must be the value of q in terms of Q to maximise the electrostatic repulsion between the two charges ?

Ans. The electrostatic repulsive force between two charge parts is

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(Q-q)}{r^2} = \frac{qQ - q^2}{4\pi\epsilon_0 \cdot r^2}$$

For force to be maximum $\frac{dF}{dq}$ must be zero.

$$\therefore \quad \frac{dF}{dq} = \frac{1}{4\pi\epsilon_0 \cdot r^2}[Q - 2q] = 0$$

$$\Rightarrow \quad Q - 2q = 0 \quad \text{or} \quad q = \frac{Q}{2}.$$

Q.33. Two small identical electrical dipoles AB and CD, each of dipole moment 'p' are kept at an angle of 120° as shown in the Fig. 1.50. What is the resultant dipole moment of this combination ? If this system is subjected to electric field ($\vec{E}$) directed along $+X$ direction, what will be the magnitude and direction of the torque acting on this ?[Delhi 2011]

Ans. As shown in Fig. 1.51, let two identical electric dipoles AB and CD are kept at an angle $\theta = 120°$, where their dipole moments have magnitude

$$|\vec{p}_A| = |\vec{p}_C| = p$$

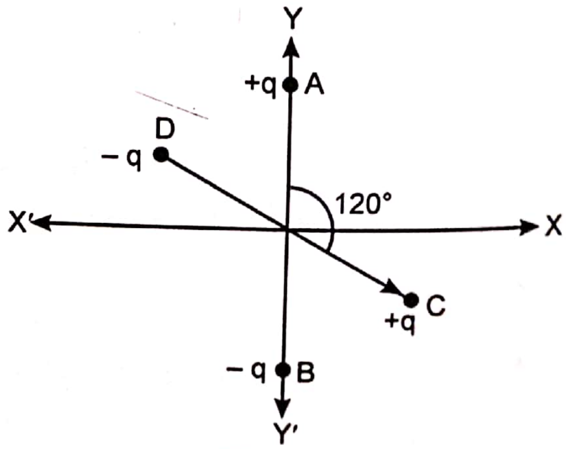

Fig. 1.50

The resultant dipole moment of the combination

$$|\vec{p}_R| = 2p\cos\frac{\theta}{2} = 2p\cos\frac{120°}{2} = 2p\cos 60° = 2p \times \frac{1}{2} = p.$$

and the resultant dipole moment subtends an angle $\frac{\theta}{2} = 60°$ from either of two dipoles $\vec{p}_A$ or $\vec{p}_B$.

Therefore $\vec{p}_R$ subtends an angle 30° from +X direction.

If the system is subjected to electric field $\vec{E}$ directed along +X direction, the torque acting on the system is

$$\vec{\tau} = \vec{p}_R \times \vec{E}$$

Thus, the magnitude of torque is $|\vec{\tau}| = pE\sin 30° = \frac{1}{2}pE$

and the torque is directed into the plane of paper *i.e.*, the torque tends to align the system along the direction of electric field $\vec{E}$.

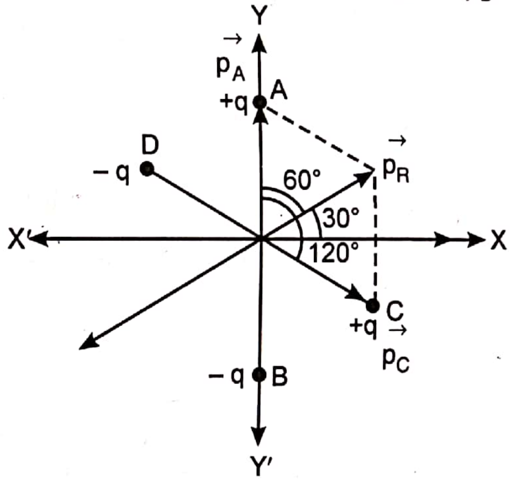

Fig. 1.51

(B – II) Short Answer Type Questions (3 marks each)

Q.34. Write Coulomb's law in vector form. Show that it is consistent with Newton's third law of motion.

Ans. Consider two point charges q_1 and q_2 separated by a distance r as shown in Fig. 1.52. Then force acting on charge q_1 due to q_2, according to Coulomb's law, is given by

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{21} \quad ...(i)$$

where $\hat{r}_{21}$ is a unit vector in the direction of q_1 from q_2 (or from q_2 to q_1).

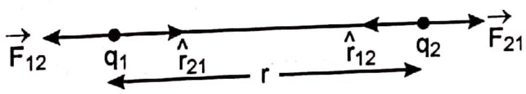

Fig. 1.52

Similarly force on charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad ...(ii)$$

where $\hat{r}_{12}$ is a unit vector from q_1 to q_2.

As unit vectors $\hat{r}_{21}$ and $\hat{r}_{12}$ have equal (unit) magnitude but are in mutually opposite directions, therefore,

$$\hat{r}_{21} = -\hat{r}_{12}$$

Hence, equation (*ii*) may be written as

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} = -\vec{F}_{21}$$

It shows that the forces exerted by the two charges on each other are equal and opposite. Thus, Coulomb's law is consistent with Newton's third law of motion.

Q.35. Define electric dipole moment. Is it a scalar or a vector ? Derive the expression for the electric field of a dipole at a point on the equatorial line of the dipole. **[A.I. 2013]**